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Reactivity Measurement Based on a New Finite-Time Convergent 2-Order Differentiator

Zhe Dong[1,*]

[1] *Institute of Nuclear and New Energy Technology, Collaborative Innovation Center of Advanced Nuclear Energy Technology, Tsinghua University, Beijing 100084, China*

ABSTRACT

Reactivity measurement is crucial in monitoring nuclear reactors. Proper estimation of neutron flux derivative is one of the key to improve the reactivity estimation based on inverse kinetics (IK) method. Signal differentiation is widely used in system monitoring and control. The key problem in differentiator design is how to obtain a satisfactory tradeoff between differentiation exactness and robustness to uncertainties and noises. By proposing a new finite-time stabilizer for 2-order integrator chain, a new finite-time-convergent 2-order differentiator with bounded estimation error is developed. This differentiator is then applied to reactivity measurement through solving the IPK equation. Numerical simulation results not only verify the theoretical results but also show its satisfactory performance.

KEYWORDS

Reactivity measurement, differentiator, finite-time stability

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1. Introduction

Reactivity estimation is crucial for the nuclear reactor monitoring and operational safety. Inverse point kinetics (IPK) method is an estimation approach that is widely implemented in the practical engineering [1]. For IPK method, the reactivity estimation is made continuously by solving the IPK equation which is a differential-integral equation containing both differentiation of the neutron flux and integration of the concentrations of the delayed neutron precursors. Some analog [2] and digital [3-7] reactivity meters based upon the IPK method were developed and implemented practically. One of the key aspects of applying the IPK method is to estimate the derivative of the neutron flux. The key aspect of applying the IPK method is to estimate the derivative of the neutron flux. Improper differentiation approach would amplify the noises, and leads to unacceptable reactivity estimation. Therefore, it is necessary to study the signal differentiation methods. In order to improve the precision of signal differentiation in solving the IPK equation, the Lagrange method [8] are introduced for higher performance of numerical differentiation. In [9], a sliding mode differentiator given in [10] was applied for the reactivity measurement, which suffers mainly from the inherent chattering effect. In this paper, a novel finite-time stabilizer of 2-order integrator chain is proposed, which guarantees that the estimation error converges to a bounded set in a finite-time. This differentiator is applied to estimate the reactivity of nuclear reactors, and numerical simulation show the feasibility and satisfactory performance of this newly-built 2-order differentiator.

2. A Finite-Time 2-Order Differentiator

Consider the global finite-time stabilization problem of the 2-order integrator chain given by

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u. \end{cases} \tag{1}$$

[*]Corresponding author, E-mail: dongzhe@mail.tsinghua.edu.cn

where $\boldsymbol{x}=[x_1, x_2]^{\mathrm{T}} \in \mathrm{R}^2$ and $u \in \mathrm{R}$.

Consider coordinate transformation

$$\boldsymbol{\xi}=\boldsymbol{\xi}(\boldsymbol{x})=\begin{bmatrix}\xi_1 & \xi_2\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}x_1 & x_2^{1/1-q}+r_1^{1/1-q}x_1\end{bmatrix}^{\mathrm{T}}, \tag{2}$$

where r_1 is a positive constant, and $0 \le q=2m/p<1/2$ with both p and m being positive odd integers.

Theorem. Suppose that signal $v(t) \in \mathrm{C}^2$ is a continuous 2-order derivable signal with

$$\left|v^{(i)}\right| \le h_i, \quad i=1,2, \tag{3}$$

where h_i (i=1,2) are both positive constants. Consider system (1) with input u given by

$$u=-\varepsilon^{-2}r_2\xi_2^{1-2q}(\bar{\boldsymbol{x}}) \tag{4}$$

where $\bar{\boldsymbol{x}}=\begin{bmatrix}x_1-v(t) & \varepsilon x_2\end{bmatrix}^{\mathrm{T}}$, ξ_2 is given by transformation (2), $\varepsilon \in (0,1)$ is the perturbation parameter

$$r_2=2^{\frac{2q}{2+q}}\sigma+2^q(1+2q)r_1^{\frac{1}{1-q}}+\frac{1-q}{1+q}\left[2^{q-1}(1+q)\right]^{\frac{1+q}{1-q}}+\frac{1+q}{1-q}r_1^{\frac{2-q}{1+q}}\left[2^{q-1}(1+2q)(1-q)\right]^{\frac{1-q}{1+q}}, \tag{5}$$

$r_1=\sigma+2$, and σ is a given strictly positive constant. Then, there $\rho>0$ so that

$$x_i-v^{(i-1)}=O\left(\varepsilon^{1/(1-iq)}\right), \quad i=1,2, \tag{6}$$

for $t>\varepsilon\Gamma$, where Γ is a strictly positive bounded scalar.

In order to prove this theorem. Some definitions and lemmas are introduced as follows.

Consider autonomous nonlinear system

$$\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x}), \quad \boldsymbol{f}(\boldsymbol{O})=\boldsymbol{O}, \quad \boldsymbol{x} \in \mathrm{R}^n, \tag{7}$$

where $\boldsymbol{f}: U_0 \rightarrow \mathrm{R}^n$ is continuous on an open neighborhood U_0 of the origin such that the solution of system (7) is unique in the forward time.

Definition 1 [9]. The equilibrium point $\boldsymbol{x}=\boldsymbol{O}$ of system (7) is called finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subset U_0$ of the origin, where the finite-time convergence means that there is a function T: $U/\{\boldsymbol{O}\} \mapsto (t_0,\infty)$ such that, for any $\boldsymbol{x}_0 \in U/\{\boldsymbol{O}\}$, the solution $\boldsymbol{x}(t)$ to system (7) with $\boldsymbol{x}(t_0)=\boldsymbol{x}_0$ as the initial condition is defined, $\boldsymbol{x}(t) \in U/\{\boldsymbol{O}\}$ for $t \in [t_0, T(\boldsymbol{x}_0))$, and

$$\lim_{t \to T(\boldsymbol{x}_0)} \boldsymbol{x}(t)=\boldsymbol{O}$$

where $T(\boldsymbol{x}_0)$ is called the settling time with respect to $\boldsymbol{x}_0$. If $U=U_0=\mathrm{R}^n$, then the origin is global finite-time stable.

Definition 2. The nonlinear control system

$$\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x})+\boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}, \quad \boldsymbol{x} \in \mathrm{R}^n, \quad \boldsymbol{u} \in \mathrm{R}^m, \tag{8}$$

is called finite-time stabilizable via continuous state-feedback if there exists a continuous feedback law $\boldsymbol{u}=\boldsymbol{u}(\boldsymbol{x})$ such that the equilibrium of the closed-loop system is finite-time stable.

The following Lemma 1 gives a sufficient condition for the origin of system (8) is globally finite-time stable.

Lemma 1 [9]. Consider autonomous system (7). If there is a real scalar $\alpha \in (0,1)$, a positive scalar c and a C^1 radially unbounded Lyapunov function $V(\boldsymbol{x})$ of the system such that

$$\dot{V}(\boldsymbol{x})+cV^{\alpha}(\boldsymbol{x}) \le 0 \tag{9}$$

holds along the trajectories of system (7) starting from any $\boldsymbol{x}_0$, then the origin point is a global finite-time stable equilibrium. Furthermore, the settling time with respect to $\boldsymbol{x}_0$ satisfies

$$T\left(\boldsymbol{x}_0\right) \leq \frac{1}{c(1-\alpha)} V^{1-\alpha}\left(\boldsymbol{x}_0\right), \tag{10}$$

where $\boldsymbol{x}_0 = \boldsymbol{x}(0) \in \mathrm{R}^n$.

Some lemmas that will be used in the sequel is introduced as follows.

Lemma 2 [8]. For any real numbers x_i, $i=1,\cdots,n$ and $0<b\leq 1$, the following inequality holds:

$$\left(\left|x_1\right|+\cdots+\left|x_n\right|\right)^b \leq \left|x_1\right|^b+\cdots+\left|x_n\right|^b. \tag{11}$$

If $b=m/p\leq 1$, where both $m>0$ and $p>0$ are odd integers, then

$$\left|x^b-y^b\right| \leq 2^{1-b}\left|x-y\right|^b, \tag{12}$$

where x, y are positive scalars.

Lemma 3 [8]. Let c, d be two positive real constants and $\gamma(x, y)>0$ be a real-valued function, where x, y are positive scalars. Then, inequality

$$|x|^c|y|^d \leq \frac{c\gamma(x,y)|x|^{c+d}+d\gamma^{-c/d}(x,y)|y|^{c+d}}{c+d}. \tag{13}$$

is well satisfied.

Proof of Theorem: Design the control input u of system (1) as

$$u=-r_2\xi_2^{1-2q}(\boldsymbol{x}) \tag{14}$$

Choose the Lyapunov function for system (1) as

$$V(\boldsymbol{x})=\sum_{k=1}^{2} W_k\left(\overline{\boldsymbol{x}}_k\right), \tag{15}$$

where

$$\overline{\boldsymbol{x}}_k=\begin{bmatrix}x_1 & \cdots & x_k\end{bmatrix}^{\mathrm{T}}, \quad k=1,2, \tag{16}$$

$$W_k\left(\overline{\boldsymbol{x}}_k\right)=W_k\left(\xi_{k-1}, x_k\right)=\int_{-\overline{r}_{k-1}\xi_{k-1}^{1-(k-1)q}}^{x_k}\left[s^{\frac{1}{1-(k-1)q}}+\overline{r}_{k-1}\xi_{k-1}\right]^{1+kq} \mathrm{d}s \tag{17}$$

$$\overline{r}_0=\xi_0=0. \tag{18}$$

From equation (17),

$$W_k\left(\overline{\boldsymbol{x}}_k\right) \leq \left|\xi_k\right|^{1+kq}\left|x_k+r_{k-1}\xi_{k-1}^{1-(k-1)q}\right| \leq 2^{(k-1)q}\xi_k^{2+q}, \tag{19}$$

$$V(\boldsymbol{x}) \leq \sum_{k=1}^{n} 2^{(k-1)q}\xi_k^{2+q}. \tag{20}$$

Differentiate function W_1 along the trajectory of model (1),

$$\begin{aligned}\dot{W}_1 &= -r_1\xi_1^2+\xi_1^{1+q}\left(x_2+r_1\xi_1^{1-q}\right)\\ &\leq -r_1\xi_1^2+2^q\left|\xi_1\right|^{1+q}\left|\xi_2\right|^{1-q}\\ &\leq -\left(r_1-\lambda\right)\xi_1^2+\frac{1-q}{1+q}\lambda^{\frac{1-q}{1+q}}\left[2^{q-1}(1+q)\right]^{\frac{1+q}{1-q}}\xi_2^2\end{aligned} \tag{21}$$

Similarly, based on Lemmas 2 and 3, differentiate W_2 along the trajectory given by (1) and (14)

$$\begin{aligned}\dot{W}_2 &\leq -r_2\xi_2^2+2^q(1+2q)\overline{r}_1\left|\xi_2\right|^{1+q}\left|\dot{\xi}_1\right|\\ &\leq -r_2\xi_2^2+2^q(1+2q)\overline{r}_1\left|\xi_2\right|^{1+q}\left|\xi_2-\overline{r}_1\xi_1\right|^{1-q}\\ &\leq -\left[r_2-2^q(1+2q)\overline{r}_1\right]\xi_2^2+2^q(1+2q)\overline{r}_1 r_1\left|\xi_2\right|^{1+q}\left|\xi_1\right|^{1-q}\\ &\leq -\left[r_2-2^q(1+2q)\overline{r}_1\right]\xi_2^2+\lambda\xi_1^2+\left\{\frac{1+q}{1-q}\lambda^{\frac{1+q}{1-q}}r_1^{\frac{2-q}{1+q}}\left[2^{q-1}(1+2q)(1-q)\right]^{\frac{1-q}{1+q}}\right\}\xi_2^2\end{aligned} \tag{22}$$

Based on inequalities (21) and (22), we then have

$$\dot{V}(\boldsymbol{x}) = \dot{W}_1(\xi_1) + \dot{W}_2(\xi_1, x_2)$$
$$\leq -(r_1 - 2\lambda)\xi_1^2 - \Big\{ r_2 - 2^q(1+2q)\overline{r_1} -$$
$$\frac{1-q}{1+q}\lambda^{\frac{1-q}{1+q}}\left[2^{q-1}(1+q)\right]^{\frac{1+q}{1-q}} - \frac{1+q}{1-q}\lambda^{\frac{1+q}{1-q}} r_1^{\frac{2-q}{1+q}}\left[2^{q-1}(1+2q)(1-q)\right]^{\frac{1-q}{1+q}} \Big\} \xi_2^2. \tag{23}$$

From inequality (23), it is feasible to set

$$r_1 = \sigma + 2\lambda, \tag{24}$$

$$r_2 = 2^{\frac{2q}{2+q}}\sigma + 2^q(1+2q)\overline{r_1} + \frac{1-q}{1+q}\lambda^{\frac{1-q}{1+q}}\left[2^{q-1}(1+q)\right]^{\frac{1+q}{1-q}} + \frac{1+q}{1-q}\lambda^{\frac{1+q}{1-q}} r_1^{\frac{2-q}{1+q}}\left[2^{q-1}(1+2q)(1-q)\right]^{\frac{1-q}{1+q}}, \tag{25}$$

where σ is a given positive constant, and then from Lemma 2,

$$\dot{V}(\boldsymbol{x}) \leq -\sigma\left(\xi_1^2 + 2^{\frac{2q}{2+q}}\xi_2^2\right) \leq -\sigma\left[\sum_{k=1}^{2} 2^{(k-1)q}\xi_k^{(2+q)}\right]^{\frac{2}{2+q}} \leq -\sigma V^{\frac{2}{2+q}}(\boldsymbol{x}). \tag{26}$$

Based on Lemma 1, the closed-loop system formed by integrator chain (1) and the feedback law given by (14), (2), and (24) is globally finite-time stable.

Define the observation error $\boldsymbol{e}$ as

$$\boldsymbol{e} = \begin{bmatrix} e_1 & e_2 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} x_1 - v & x_2 - \dot{v} \end{bmatrix}^{\mathrm{T}}, \tag{27}$$

and define

$$\begin{cases} \boldsymbol{z} = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^{\mathrm{T}}, \\ z_k = \varepsilon^{k-1} e_k, \quad k = 1, 2. \end{cases} \tag{28}$$

From (1), (27) and (28), observation error $\boldsymbol{e}$ satisfies

$$\begin{cases} \dfrac{\mathrm{d}z_1}{\mathrm{d}\tau} = z_2, \\ \dfrac{\mathrm{d}z_2}{\mathrm{d}\tau} = -r_2\xi_2^{1-2q}(\boldsymbol{z} + \overline{\boldsymbol{v}}) - \dfrac{\mathrm{d}^2 v}{\mathrm{d}\tau^2}, \end{cases} \tag{29}$$

where

$$\tau = \frac{t}{\varepsilon}, \tag{30}$$

$$\overline{\boldsymbol{v}} = \begin{bmatrix} 0 & \dfrac{\mathrm{d}v}{\mathrm{d}\tau} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & \varepsilon\dot{v} \end{bmatrix}^{\mathrm{T}}. \tag{31}$$

Choose the Lyapunov function of system (29) as

$$V(\boldsymbol{z}) = \sum_{k=1}^{2} W_k(\boldsymbol{z}), \tag{32}$$

where function W_k is defined by (17), and k=1, 2.

By differentiating $V(\boldsymbol{z})$ along the trajectory of system (29), and based on Lemma 2 and inequality (26), we can derive that

$$\frac{\mathrm{d}V(\boldsymbol{z})}{\mathrm{d}\tau} \leq \left.\frac{\mathrm{d}V(\boldsymbol{z})}{\mathrm{d}\tau}\right|_{v\equiv 0} + \left|\xi_2(\boldsymbol{z})\right|^{1+2q}\left|v^{(2)}\right| + r_2 2^{2q}\left|\xi_2(\boldsymbol{z})\right|^{1+2q}\left|\xi_2(\boldsymbol{z}+\overline{\boldsymbol{v}}) - \xi_2(\boldsymbol{z})\right|^{1-2q}$$
$$\leq -\frac{\sigma}{2}\sum_{k=1}^{2} 2^{\frac{2(k-1)q}{2+q}}\xi_k^2(\boldsymbol{z}) + \sum_{k=1}^{2} M_k h_k^{\frac{2}{1-kq}}\varepsilon^{\frac{2k}{1-kq}} \tag{33}$$

where

$$M_1 = (1-2q)2^{\frac{2q(1-2q)}{(2+q)(1+2q)}}\sigma^{\frac{1-2q}{1+2q}}\left[r_2(n-1)(1+nq)2^{2q+1}\right]^{\frac{1+2q}{1-2q}}, \tag{34}$$

$$M_2 = (1-2q)2^{\frac{2q(1-2q)}{(2+q)(1+2q)}+\frac{4q}{1-2q}}\sigma^{\frac{1-2q}{1+2q}}(1+2q)^{\frac{1+2q}{1-2q}}. \tag{35}$$

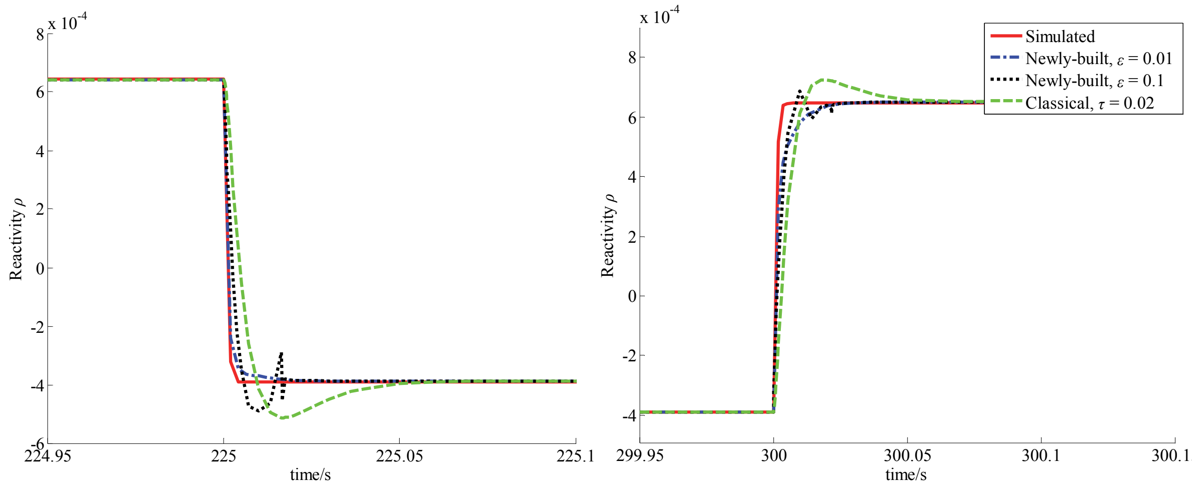

Fig. 1. Simulation results: τ is the inertial of classical differentiator

For the initial point z_0 in the outside of bounded set

$$\Xi = \left\{ z \in \mathrm{R}^2 \left| \; \frac{\sigma}{4} \sum_{k=1}^{2} 2^{\frac{2(k-1)q}{2+q}} \xi_k^2(z) \le \sum_{k=1}^{2} M_k \varepsilon^{\frac{2k}{1-kq}} \left| v^{(i)} \right|^{\frac{2}{1-kq}} \right. \right\}, \tag{36}$$

and from (33), we have

$$\frac{\mathrm{d}V(z)}{\mathrm{d}\tau} \le -\frac{\sigma}{4} V^{\frac{2}{2+q}}(z), \tag{37}$$

which means that state-vector z is in set Ξ if $t > \varepsilon\Gamma$ with

$$0 < \Gamma \le \frac{8+4q}{q\sigma} V^{\frac{q}{2+q}}(z_0). \tag{38}$$

Furthermore, from coordinate transformation (2) there exist positive constants λ_k (k=1, 2) so that

$$|e_k| \le \lambda_k \varepsilon^{\frac{1}{1-kq}} h_k^{\frac{1-(k-1)q}{1-kq}}, \quad k = 1, 2, \tag{39}$$

which means that relationship (6) can be well satisfied. This completes the proof of Theorem.

3. Application to Reactivity Measurement with Simulation Results

The reactivity estimation is given by the following inverse kinetic equation

$$\rho = \Lambda \frac{\mathrm{d}\ln(n_\mathrm{r})}{\mathrm{d}t} + \beta - \sum_{i=1}^{6} \frac{\beta_i}{n_\mathrm{r}} e^{-\lambda_i t} \left[n_{\mathrm{r}0} + \lambda_i \int_{t_0}^{t} e^{\lambda_i \tau} n_\mathrm{r}(\tau) \mathrm{d}\tau \right], \tag{40}$$

where n_r is normalized neutron flux with initial value $n_{\mathrm{r}0}$, Λ is the prompt neutron lifetime, λ_i and β_i is respectively the precursor decay constant and fraction of the ith group delayed neutron. From (40), the derivative of $\ln(n_\mathrm{r})$ is the key in reactivity estimation.

In order for verification the newly-built differentiator in reactivity measurement. Consider the following case for simulation. Initially, the reactor is steady in 10% full power (FP), and at 200s, a square wave of reactivity with amplitude of 15%β and period of 50s is induced periodically. The new finite-time convergent 2-order differentiator and the classical 1-order differentiator are used. Here, the transfer function of the 1-order classical differentiator is

$$G_\mathrm{D}(s) = \frac{s}{\tau s + 1}, \tag{41}$$

and τ is the inertial constant. In this simulation, parameters σ and q of 2-order finite-time differentiator given by (1) and (4) are chosen to be σ=10 and q=2/5, and different values of ε are adopted. The inertial constant τ of classical differentiator (41) is chosen to be τ=0.02, which has no relationship with the parameters of newly-built finite-time differentiator. The simulation results with comparison between 2-order finite-time differentiator and classical differentiator are shown in Fig. 1.

From Fig. 1, the reactivity estimation precision is higher if ε is smaller, which is in accordance with relationship (6), and thus verifies the theoretic results given by Theorem. Comparing to classical differentiator, thanks to nonlinear feedback law (14), the newly-built differentiator composed by (1) and (14) guarantees higher transient and steady performance.

4. Conclusion

Motivated by importance of signal derivatives in providing high performance reactivity measurement, a new finite-time convergent 2-order differentiator is proposed, whose key idea is to transfer the problem of differentiator to the problem of finite-time stabilization of a 2-order integrator chain. It is analyzed theoretically that the estimation error of both the signal and its derivative converges to a bounded set in a finite-time. This novel differentiator is applied to solving the IPK equation for reactivity measurement. Numerical simulation not only verifies the correctness of the theoretical results but also shows the satisfactory performance of this differentiator. One of the future works is to extend this 2-order differentiator to high-order version.

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