E-Journal of Advanced Maintenance Vol.2 (2010/2011) 160-167
Japan Society of Maintenology

Numerical Simulation of Residual Stress Measurement with Acoustic Wave

Cuixiang PEI* and Kazuyuki DEMACHI

Department of Nuclear Engineering and Management, University of Tokyo, Tokyo 113-8656, Japan

ABSTRACT

In this work, a FEM (finite element method) simulation program was used in the residual stress measurement with acoustic wave. The possibility of assessing the residual stress, by using an EMAT (electromagnetic acoustic transducer) receiver for precise measurement of the acoustoelstic effect on LCR wave, was investigated by simulation method.

KEYWORDS

Residual stress, Acoustoelastic effect, Ultrasonic NDE, FEM, EMAT

ARTICLE INFORMATION

Article history:
Received 19 August 2010
Accepted 1 February 2011

1. Introduction

The residual stress in some mechanical structures increases the likelihood of fatigue cracks, stress corrosion cracks and environmental induced material degradation. The ability to evaluate the residual stress would substantially increase the accuracy of structure life estimation and the security of operation. Several nondestructive testing methods, such as X-Ray diffraction techniques, ultrasonic wave techniques and magnetic effect analytical methods, have been applied in the residual stress measurement. As the high permeability of ultrasonic wave, the ultrasonic wave method can be used to evaluate the residual stress in the interior of the structure as well as at the surface. Besides, being portable and cheap to undertake, this method is well suited to routine inspection procedures and industrial studies of large components [1].

The ultrasonic methods for residual stress measurement are mainly based on the acoustoelastic effect related to the variation of ultrasonic wave velocity and change in the polarization of Rayleigh wave. Presently, the residual stress measurement with ultrasonic wave is mainly studied with experiment method [2,5]. And simultaneously, the numerical simulation method can provide another useful tool for the research of stress measurement with ultrasonic wave. In this work, the possibility of numerical simulation of residual stress measurement based on the acoustoelastic effect was investigated with FEM.

Now in practical application, the LCR (refracted longitudinal) wave is usually used to measure the subsurface residual stress in components. In this paper, the possibility of assessing the stress status, by using an EMAT (electromagnetic acoustic transducer) receiver for precise measurement of LCR (refracted longitudinal) wave, was investigated by simulation.

2. Numerical Simulation Method

2.1 Analytical model for acoustoelastic effect in solids

An ultrasonic wave through a stressed body would give rise to further stresses, and the traditional theory of acoustoelasticity applied in the unstressed medium cannot be used here. Instead, based on the deformation process, Duquennoy et al. defined a theory of three state of a body, shown in Fig.1 [3,4]. A solid body undergoes a series of deformations from a stress free state to a static deformation

*Corresponding author, E-mail: peicx@nuclear.jp
† Present address : 2-11-16, Yayoi, Bunkyo-ku, Tokyo 113-0032, Japan

or a dynamic deformation. The position vector ξ defines the position of a point in the natural state of zero stress and zero strain. X defines the position of a point in the initial state when the body undergoes a static deformation due to residual stress during the manufacturing processes or due to the applied stresses. Similarly x defines the position in the final states when a dynamic deformation such as the ultrasonic wave through the body. The displacement of a point from one state to another can be described mathematically as:

$$u^i(\xi) = X - \xi;\, u^f(\xi) = x - \xi;\, u(\xi) = x - X = u^f - u^i$$

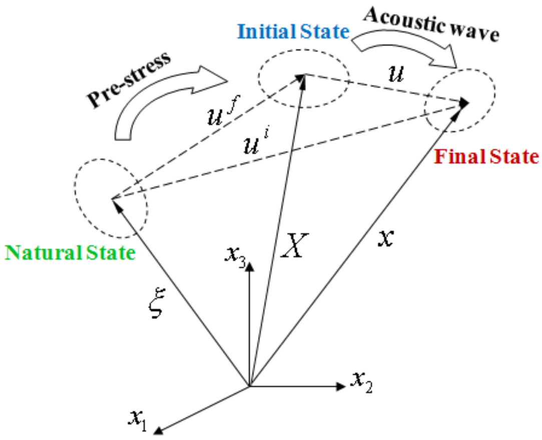

Fig. 1. Coordinates for material point in three states

Based on the three-state theory, the controlling equation of the acoustic wave in a solid with initial stress can be written as

$$\left(\delta_{IK}\sigma^i_{JL} + \hat{D}_{IJKL}\right)\frac{\partial^2 u_K}{\partial X_J X_L} - \gamma\frac{\partial u_I}{\partial t} + f = \rho^0(1-\varepsilon^i_{NN})\frac{\partial^2 u_I}{\partial t^2} \tag{1}$$

Where σ^i is the Cauchy stress tensor in the initial state, ε^i the initial strain tensor induced by the residual stress and ρ^0 the mass density in the natural state, γ is the acoustic damping coefficient, f is the external force loading on the body. And for isotropic material the elastic constant can be expressed as

$$\begin{aligned}\hat{D}_{IJKL} = {} & \lambda\delta_{IJ}\delta_{KL} + \mu(\delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK}) + [(-\lambda + v_1)\delta_{IJ}\delta_{KL} \\ & + (-\mu + v_2)(\delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK})]\varepsilon^i_{NN} + 2(\lambda + v_2)(\varepsilon^i_{IJ}\delta_{KL} + \varepsilon^i_{KL}\delta_{IJ}) \\ & + 2(\mu + v_3)(\varepsilon^i_{IK}\delta_{JL} + \varepsilon^i_{IL}\delta_{JK} + \varepsilon^i_{JK}\delta_{IL} + \varepsilon^i_{JL}\delta_{IK})\end{aligned} \tag{2}$$

Where λ and μ are the well known Lame constants, $v_i(i=1,2,3)$ are the third-order elastic constants (TOE constants), and δ_{IJ} is Kronecker delta.

2.2. Numerical Model

In order to demonstrate the feasibility of this simulation method, a 3D model with initial homogeneous uniaxial stress σ_x (the stress σ_{xx}, uniaxially directed along the x axis) was used for the simulation of acoustoelasticity, shown as in Fig.2. A PZT angle transducer was used to generate surface wave, and an EMAT probe was used to receive the time behavior of the wave signal for different initial stress σ_x. As shown in table 1[6,8], two kinds of material have been used in this paper. Where, the sensitivity constant k_c is defined as a relative change in wave velocity per unit change in stress. When the longitudinal wave velocity through a wedge is $c_w = 2720m/s$, based on Snell's law, the incidence angle should be set at about $\theta_w = 26^\circ$ to generate LCR wave in the model.

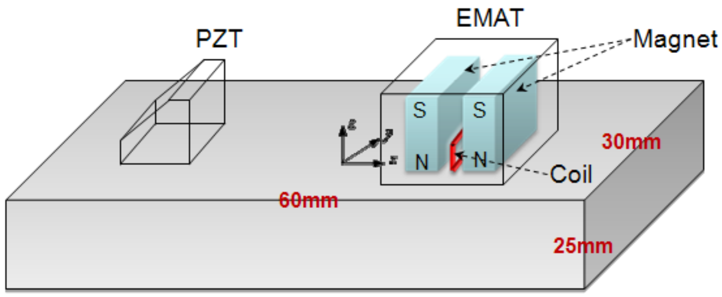

Fig. 2. A pitch-catch setup model in simulation

Table 1 Material parameter of the model

Material	$\rho / kg \cdot m^{-3}$	$\sigma_e / S \cdot m^{-1}$	λ / Pa	μ / Pa
Al Alloy D54s	2719.0	38.2E6	49.1E9	26.0E9
Rail Steel	7800.0	1.1E6	115.8E9	79.9E9
Material	v_1 / Pa	v_2 / Pa	v_3 / Pa	$k_c / (MPa)^{-1}$
Al Alloy D54s	-379.0E9	-198.0E9	-80.0E9	-8.79E-5
Rail Steel	36.0E9	-266.0E9	178.5E9	-1.214E-5

2.3. Numerical simulation method for acoustic wave

When there is residual stress in the material, the initial strain ε_{IJ}^i would be induced. Based on the relationship between the stress and strain in isotropic material, the initial strain can be calculated by

$$\varepsilon_{IJ}^i = \frac{1+\upsilon}{E}\sigma_{IJ} - \delta_{IJ}\frac{\upsilon}{E}\sigma_{kk} \tag{3}$$

$$E = \frac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}, \qquad \upsilon = \frac{\lambda}{2(\lambda+\mu)}$$

Substituting Eq. (3) into Eq. (2), the elastic constant related to the residual stress can be decided. According to FEM, eq. (1) can be calculated by solving the discrete wave motion eq. (4)

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F\} \tag{4}$$

where $[M] = \sum_e [M^e]$ denotes the mass matrix, $[C] = \sum_e [C^e]$ the damping matrix, $[K] = \sum_e [K^e]$ the stiffness matrix, $\{U\}$ the nodal displacement and $\{F\}$ the force vector.

$$[M^e]=\rho\int_{V^e}[N]^T[N]dV$$
$$[C^e]=\gamma\int_{V^e}[N]^T[N]dV$$
$$[K^e]=\int_{V^e}[B]^T[\hat{D}_{IJKL}][B]dV$$

$[N]$ is the shape function and $[B]$ the strain matrix of the element used in the FEM model. In order to solve the differential function, Eq. (4) is rearranged as into the following equation by using the explicit integration method in time-domain[7]

$$\begin{cases}\{\dot{U}\}_{t+\Delta t}=\{\dot{U}\}_{t-\Delta t}-2\Delta t[M]^{-1}[C]\{\dot{U}\}_t-2\Delta t[M]^{-1}[K]\{U\}_t+2\Delta t[M]^{-1}\{F\}_t\\ \{U\}_{t+\Delta t}=\{U\}_t+\dfrac{\{\dot{U}\}_{t+\Delta t}+\{\dot{U}\}_t}{2}\Delta t\end{cases}\tag{5}$$

2.4. Modeling of PZT angle transducer

To simplify the problem, the modeling approach of the PZT angle transducer presented here focuses on the transmission of mechanical wave between the transducer and the test-piece. As shown in Fig. 3, the transmission of mechanical wave is modeled as a distribution of force $f_i(t)$, distributed only on the excited portion of the test-piece's boundary surface. The function $f_i(t)$ is given as

$$f_i(t)=\begin{cases}A\sin(2\pi f t_i); & 0\le t_i\le T\\ 0; & t_i<0 \;\; or \;\; t_i>T\end{cases}\tag{6}$$
$$t_i=t+\frac{(i-1)}{(N-1)}\frac{d\sin\theta_w}{c_w}$$

where A is the amplitude of the exciting signal, T the pulse width of the exciting signal , f the central frequency of the exciting signal and N the number of contributing nodes.

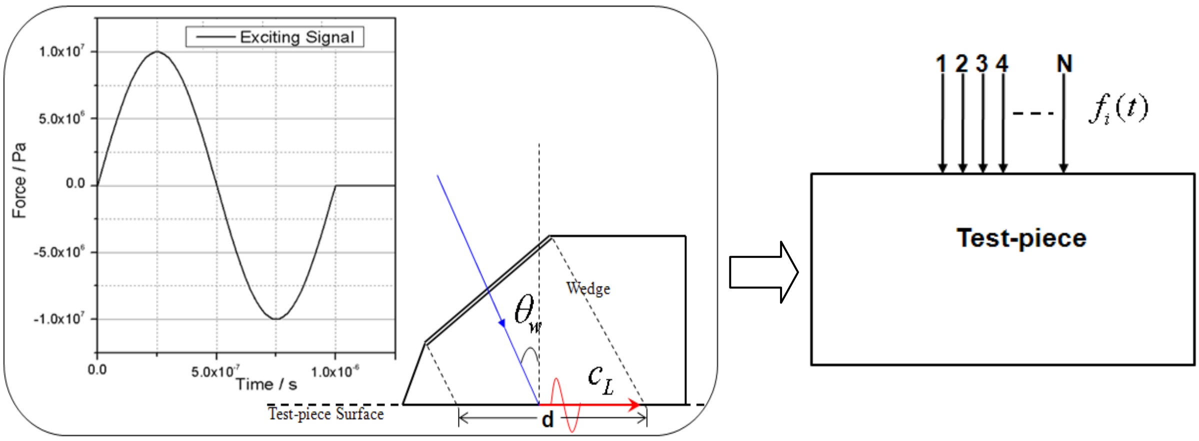

Fig. 3. Modeling of the PZT angle transducer

2.5. Numerical method for EMAT receiver

A typical configuration for an EMAT receiver of acoustic waves in a test-piece is shown in Fig. 2. It is usually composed of a static magnet and a set of coils (named pick-up coil). When the ultrasonic wave occurred in the test-piece under the EMAT receiver, the vibration by the ultrasonic wave would interact with the static magnetic flux density of the static magnet $\mathbf{B}_0$ to yield transient eddy current $\mathbf{J}_e$ in the metal. $\mathbf{J}_e = \sigma_e \mathbf{v} \times \mathbf{B}_0$, $\mathbf{v}$ is the velocity of a particle, σ_e is the conductivity of the metal. And a micro current would be induced by the electromagnetic induction effect. The magnetic flux density $\mathbf{B}_0$ of a permanent magnet can be calculated by equivalent magnetic charge approach [7]. The most direct way to calculate the pick-up signals of the EMAT receiver is to use the Biot-Savart's law, and the voltage signal in pick-up coil can be written as,

$$V_e = \sum_{i=1}^{N} \frac{\partial}{\partial t} \oint_{\Gamma_i} \mathbf{A}_e^f \cdot d\mathbf{l} = \sum_{i=1}^{N} \frac{\partial}{\partial t} \oint_{\Gamma_i} \left[\iiint_{cond} \frac{\mathbf{J}_e}{r} dv \right] \cdot d\mathbf{l} \tag{7}$$

where Γ_i is boundary of surface S_i and r is the distance from a current source point in the conductor to a point in the pick-up coil.

3. Numerical Simulation Results

The ultrasonic wave was excited by one damped cycle of a 1 MHz sinusoidal. To give a preliminary judgment of the validity of this method, the simulation result of the ultrasonic wave field in a cross section at a given time in the Al alloy model is shown in Fig. 4. The time behavior of the EMAT coil voltage is plotted in Fig. 5. As the EMAT receiver was set at 40 mm away from the PZT transducer, the velocity of the LCR wave is calculated which is about 6000 m/s. This simulation result is agreed with the theoretical value.

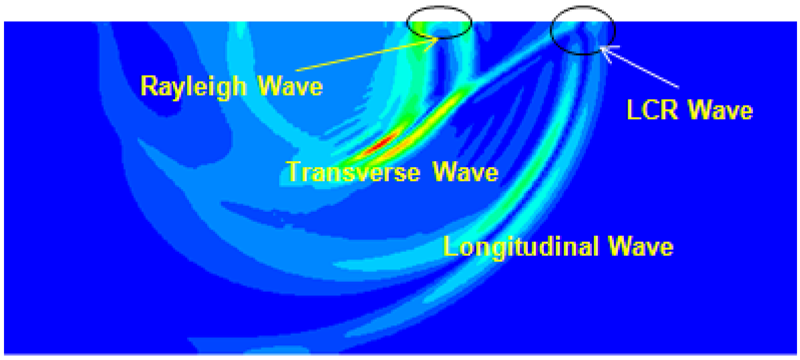

Fig. 4. Ultrasonic wave field excited by PZT

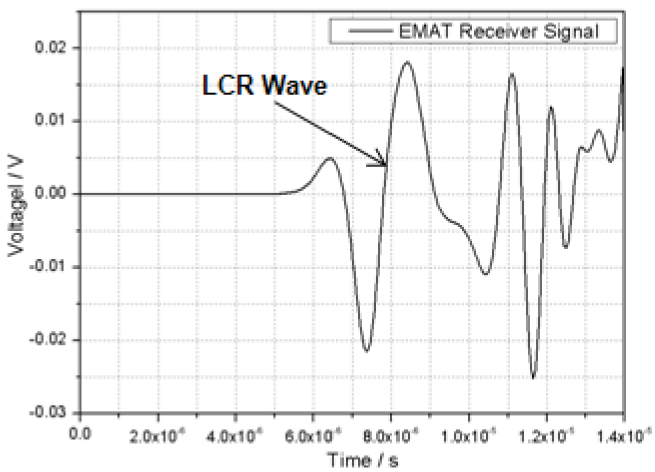

Fig. 5. LCR wave received by EMAT

In Fig. 6 and Fig. 7, the close-up of the LCR wave signals for two initial stresses in Al Alloy and rail steel is provided. The acoustoelastic effect with a small phase delay induced by the stress can be observed. It can be seen that the variation in LCR wave velocity due to acoustoelastic effect in rail steel is much smaller than that in Al alloy.

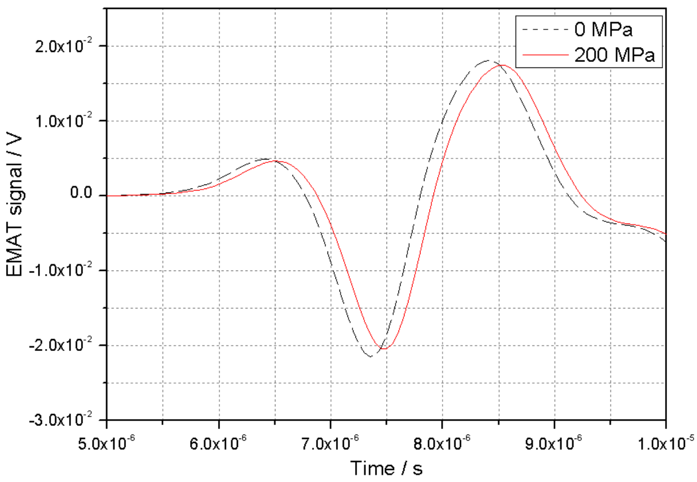

Fig. 6. Acoustoelastic effect in Al alloy

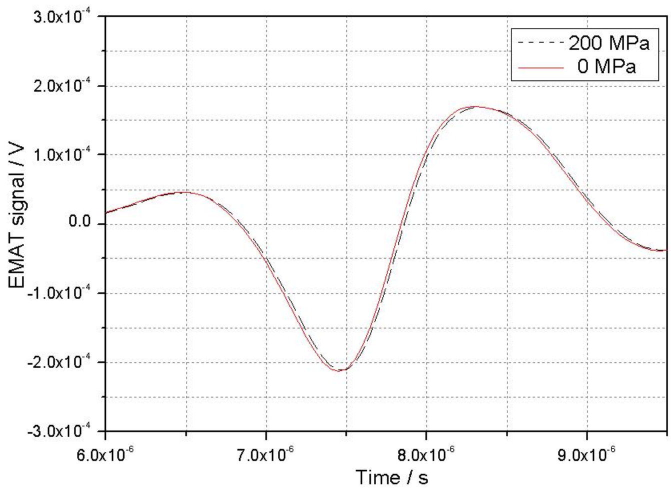

Fig. 7. Acoustoelastic effect in rail steel

It can be seen that the time delay induced by the stress is very small in Fig.6 and Fig.7. So it is hard to evaluate the relative change in wave velocity directly. To calculate the relative change in wave velocity indirectly, a precise method based on the cross correlation algorithm was executed as following.

A. Firstly, the cross correlation coefficient of the two wave signals at the same measuring position with and without residual stress was calculated by delaying the start time of one signal with Eq.(8)

$$r(i*dt)=\frac{\sum_{k=1}^{N}\left[\left(x(k)-mx\right)\cdot\left(y(k-i)-my\right)\right]}{\sqrt{\sum_{k=1}^{N}\left(x(k)-mx\right)^2}\sqrt{\sum_{k=1}^{N}\left(y(k-i)-my\right)^2}} \quad (8)$$

where dt is the interval between the time resolution, r the correlation coefficient at delay i*dt, x and y are the two series of wave signal, mx and my are the means of the two series. Fig. 8 shows an example of the result of the cross correlation coefficient of two wave signals. And the delay time Δt can be precisely decided when the correlation coefficient reaches the maximum.

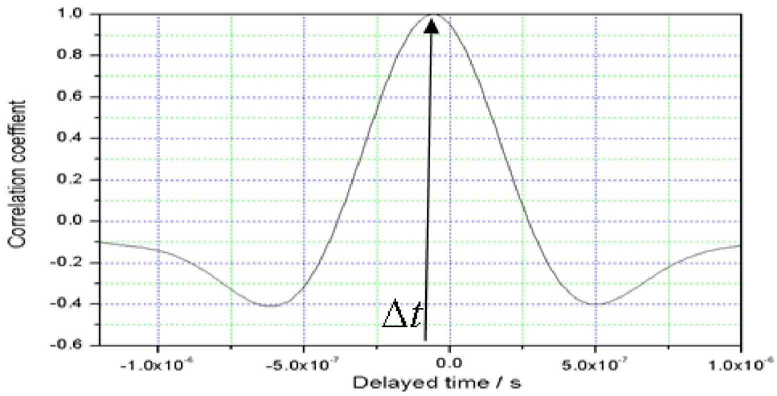

Fig. 8. Example of the calculation result of the cross correlation between two signals

B. The relative change of the wave velocity can be calculated with Eq. (9)

$$\frac{\Delta c}{c_0} = \frac{c_1 - c_0}{c_0} = -\frac{c_0 \Delta t}{s + c_0 \Delta t} \tag{9}$$

where c0 denotes the wave velocity without residual stress, c1 the velocity with residual stress, s the distance between the two transducers and Δc the velocity change due to the stress. To overcome the artificial error in the calculation of wave velocity, the c0 was set as the theoretical value of wave velocity without residual stress here.

Based on the above method, the relative changes in LCR wave for different residual stress were simulated, as shown in Fig.9. It can be seen that the simulation results show a very good agreement with the theoretical values.

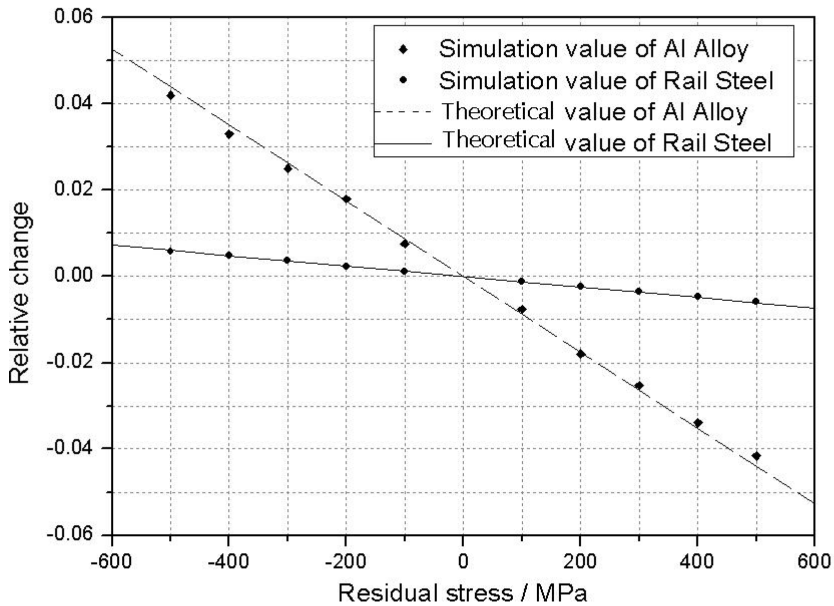

Fig. 9. Relative change of LCR wave's velocity in Al Alloy and rail steel

4. Conclusion

In this paper, a numerical code is developed for numerical simulation of acoustoelatic effect in pre-stressed media. The possibility of assessing the stress status, by using the EMAT for precisely measuring the LCR wave, was investigated. The numerical results showed a very good agreement with the theoretical values.

Acknowledgement

This work was supported by China Scholarship Council (CSC).

References

[1] P. J. Whithers and H. K. H. Bhadeshia, Overview Residual stress Part1- Measurement techniques, Material Science and Technology 17 (2001), 355-365.
[2] Development of non-contact stress measurement system during tensile testing using the electromagnetic acoustic transducer, NDT&E 39 (2006), 299-303.
[3] Michael Junge, Jianmin Qu, Relationship between Rayleigh wave polarization and state of stress, Ultrasonics 44 (2006), 233-237.
[4] Dequennoy M. Ouaftouh M., Ultrsonic evaluation of stresses in orthotropic materials using Rayleigh waves, NDT&E 32 (1999), 189-199.
[5] Shailesh Gokhale, Determination of applied stress in rails using the acoustoelastic effect of ultrasonic waves, Master's thesis, Texas A&M University, 2007.
[6] Michael Junge, Measurement of applied stress using polarization of Rayleigh waves. Master's thesis, School of Civil and Environmental Engineering, Georgia Institute of Technology, 2003.
[7] Cuixiang Pei, Zhenmao Chen, Development of simulation method for EMAT signals and application to TBC inspection, International Journal of Applied Electromagnetics and Mechanics 33 (2010), 1077-1085.
[8] R.T. Smith, R. Stern, and R.W.B Stephens. Third-order elastic module of polycrystalline metals from ultrasonic velocity measurements. Journal of Acoustical Society of American 40(5) (1966), 1002-1008.
[9] Don E. Bray, Wei Tang, Subsurface stress evaluation in steel plates and bars using the LCR ultrasonic wave, Nuclear Engineering and Design 207 (2001), 231-240.